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TWO-DIMENSIONAL CALCULATION OF THE PARAMETERS OF A STEAM-AIR MIXTURE IN A FILM-TYPE HEAT AND MASS EXCHANGER

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A mathematical model for two-dimensional calculation of the temperature and density of the steam in a steam-air mixture which rises between two water films flowing down adiabatic shields has been proposed. The parameters of the films have been calculated in a one-dimensional approximation. The results of calculating the parameters of air according to the one-dimensional and two-dimensional models of the processes of heat and mass transfer have been compared. The range of applicability of the former to a film-type heat and mass exchanger has been determined.

Introduction. Problems on intensification of the processes in heat and mass exchangers and on decreasing the geometric parameters of the structures have become increasingly urgent in developing modern heat and mass exchangers. In this connection, in mathematical modeling of the transfer processes in heat- and mass-exchange equipment, one must take into account the spatial distribution of the physical parameters of heat-transfer agents. The need for a three-dimensional (two-dimensional, as a last resort) description of the transfer processes in industrial apparatuses does not generate distinctive discussions. In particular, the same problems exist in developing structures of air humidifiers that are used in industry and agriculture. The optimum temperature of the air going out of a humidifier lies in the interval 20–22^{o}C, while its relative humidity amounts to 30–60%.

In this work, we have made a comparative analysis of the one-dimensional and two-dimensional descriptions of the transfer processes in the gas phase of a humidifier and have presented the iterative method of calculation of its parameters. Analogous processes of transfer are observed in many film-type heat exchangers.

Figure 1 shows a diagram of the main unit of a film-type heat and mass exchanger in which contact heat and mass exchange occurs between the water and the air. We investigate the case where the counterflow of the heat-transfer agents is observed (this case is the most difficult to model). From the mathematical viewpoint, we consider the boundary-value problem for a system of differential equations where both ordinary differential equations and partial equations are employed in two-dimensional modeling.

It is clear that the characteristics of the air and the water going out of the film-type heat and mass exchanger are affected by many parameters, the most significant of which are: specific mass flow rate of the water Q_w, initial temperatures of the water film T_{w0} and the air T_{a0}, relative humidity of the air ψ, and velocity of the air v_a between the shields. A significant role is also played by the geometric parameters: height of the shields H and distance between them d. We outline the obtained one-dimensional mathematical model of the processes of heat and mass exchange between the steam-air mixture and two water films [1, 2].

One-Dimensional Mathematical Model. We guide the z axis vertically downward and bring its origin into coincidence with the upper edge of the shields (Fig. 1).

The system of equations describing the processes of heat and mass exchange of a vertically flowing film with an ascending steam-air mixture, upon the averaging of the field of the temperature and velocity of the film and the temperature and density of the steam in the steam-air mixture, involves the equations describing the changes in:

(a) the thickness of the film $h(z)$ due to its evaporation

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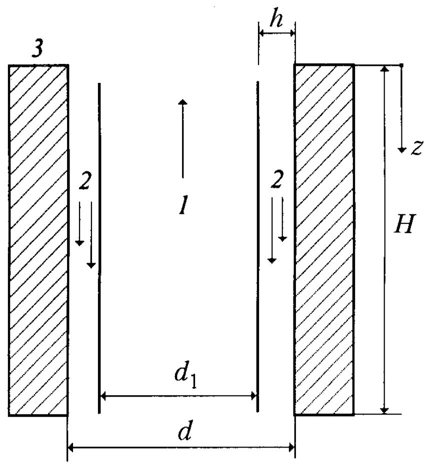

Fig. 1. Diagram of motion of air 1 and water 2 over the vertical shields of the film-type heat and mass exchanger 3.

$$\frac{dh\,(z)}{dz} = -\frac{\gamma\,(\mathrm{Re})\,(\rho_s\,(T_w\,(z)) - \rho\,(z))}{\rho_w v_w}\,; \tag{1}$$

(b) the water temperature average over the film cross section $T_w(z)$ due to the contact with warm air and to evaporation

$$\frac{dT_w\,(z)}{dz} = \frac{\alpha\,(\mathrm{Re})\,(T_a\,(z) - T_w\,(z)) - r\gamma\,(\mathrm{Re})\,(\rho_s\,(T_w\,(z)) - \rho\,(z))}{c_w\,\rho_w h\,(z)\,v_w}\,; \tag{2}$$

(c) the steam-air-mixture temperature average over the cross section between the shields $T_a\,(z)$

$$\frac{dT_a\,(z)}{dz} = -\frac{2\alpha\,(\mathrm{Re})\,(T_w\,(z) - T_a\,(z))}{v_a d_1\,\rho_a c_a}\,; \tag{3}$$

(d) density of the steam $\rho(z)$ in the air

$$\frac{d\rho\,(z)}{dz} = -\frac{2\gamma\,(\mathrm{Re})\,(\rho_s\,(T_w\,(z) - \rho\,(z))}{v_a d_1}\,. \tag{4}$$

The system of equations (1)–(4) has been integrated numerically with the following boundary conditions: for $z = 0$

$$h\,\big|_{z=0} = h_0\,, \quad T_w\,\big|_{z=0} = T_{w0}\,; \tag{5}$$

for $z = H$

$$T_a\,\big|_{z=H} = T_{a0}\,, \quad \rho\,\big|_{z=H} = \rho_s\,(T_{a0})\,\psi = \rho_0\,. \tag{6}$$

We note that the cross section for passage of the air between the shields d_1 depends on the water-film thickness h, which in turn is a function of the specific wetting density Q_w:

$$d_1 = d - 2h\,(Q_w)\,. \tag{7}$$

The heat-exchange coefficient $\alpha(\mathrm{Re})$ in Eqs. (2) and (3) has been taken from [3] and it is equal to

$$\alpha = \frac{0.324\ \mathrm{Re}^{0.5}\ \mathrm{Pr}^{0.33} \lambda_a}{x} . \tag{8}$$

The Reynolds number with allowance for the geometry of the problem is determined as [3, 4]

$$\mathrm{Re} = \frac{x \rho_a \left(v_a + v_w\right)}{\mu_a} . \tag{9}$$

The variable x in (8) and (9) is determined as $x = H - z$ and it is employed for description of the air mixture in the wetting device (sprayer); $v_a + v_w$ is the velocity of air flow about the flowing-down film of water.

To determine the dependence of γ in Eqs. (1), (2), and (4) we have used the analogy between the dependence of the heat-exchange ($\mathrm{Nu} = \alpha x / \lambda$) and mass-exchange ($\mathrm{Sh} = \gamma x / D$) numbers on the Reynolds number. Then for the coefficient of mass exchange in laminar flow about a thin water film we have the following expression:

$$\gamma = \frac{0.324\ \mathrm{Re}^{0.5}\ \mathrm{Pr}^{0.33} D}{x} . \tag{10}$$

In the calculations, we took into account the dependences of the diffusion coefficients of the steam in the air D and of other thermophysical parameters on the temperature [5].

As is well known [6], the regime of flow of a film depends on the specific wetting density. The flow regime is laminar wave-free for low values of the wetting density, but as the wetting density increases, the velocity of flowing of the laminar film on whose surface waves appear increases; the wavelength decreases with increase in the flowing velocity. In such a regime of film flow on the adiabatic wall, it is very efficient to employ the average temperature in the film cross section.

The average velocity of flowing of the water film, which is involved in (1) and (2), is written as [4, 7]

$$v_w = \left(\frac{g}{2\nu_w}\right)^{1/3} \left(\frac{Q_w}{\rho_w}\right)^{2/3} . \tag{11}$$

The air velocity v_a in the one-dimensional model is a coordinate-independent free parameter.

Boundary-value problem (1)–(6) for the system of nonlinear ordinary differential equations (1)–(4) was solved by the "shooting" method [8]. For numerical solution of (1)–(4) we used the Runge–Kutta scheme. The accuracy of computations was monitored using the discrepancy criterion

$$\Sigma = \sqrt{\left(\frac{T_a(H) - T_{a0}}{T_{a0}}\right)^2 + \left(\frac{\rho(H) - \rho_0}{\rho_0}\right)^2} . \tag{12}$$

We stopped the numerical calculation when the condition $\Sigma < 10^{-4}$ was satisfied.

The calculations according to the film model have shown that the decrease in the air temperature and the increase in the steam density in the system in question nonlinearly depend on the shield height. This is due to the saturation of the passing air with steam, which makes the processes of heat and mass exchange less intense in evaporation of the water film.

A qualitative analysis of the mathematical model of film cooling of water, which enables one to obtain approximate formulas, has been made in [1]. In particular, it was shown that the density of the steam ρ in the outgoing air can be represented as

$$\rho \sim \frac{D}{d} \left[\frac{\rho_a H \left(1 + v_w / v_a\right)}{\mu_a v_a}\right]^{0.5} \rho_s \left(T_{w0}\right) . \tag{13}$$

From (13), it is seen that the water flow rate affects the steam density in terms of the velocity ratio v_w/v_a and that the main factors determining the density of the steam in the steam-air mixture going out of the film-type heat and mass exchanger are the initial temperature of the water and the parameter *D/d*.

The analogous evaluation for the temperature of the steam-air mixture leads to the following approximate expression:

$$T_a \sim \frac{\lambda_a}{dc_a \rho_a}\left[\frac{H\rho_a (1 + v_w/v_a)}{\mu_a v_a}\right]^{0.5} [T_{w0} - T_{a0}] . \tag{14}$$

Equation (14) shows that the temperature of the air T_a going out of the film-type heat and mass exchanger is in direct proportion to the difference of the initial temperatures of the water and the ambient air and in inverse proportion to the distance between the shields and it depends on the specific flow rate of the water in terms of the parameter v_w.

We discuss the problem of applicability of the one-dimensional model to description of the system in question. For example, the characteristic time of the process of transfer of heat is equal to $\tau_\perp \sim L^2/a$ in the transverse direction and to $\tau_\| \sim H/v$ in the longitudinal direction. For efficient employment of the one-dimensional approximation it is required that the condition $\tau_\perp << \tau_\|$ be satisfied. The analogous inequality is written for the characteristic times of diffusion of the steam in the air between the shields in the longitudinal and transverse directions. Thus, the criteria of efficient employment of the one-dimensional approximation for this problem have the following form:

for the transfer of heat in the water film

$$\frac{h^2 v_w}{a_w H} << 1 , \tag{15}$$

for the transfer of heat in the flow of the steam-air mixture

$$\frac{d_1^2 v_a}{a_a H} << 1 , \tag{16}$$

for the diffusion of the steam in the flow of the steam-air mixture

$$\frac{d_1^2 v_a}{DH} << 1 . \tag{17}$$

Let us perform numerical evaluations, employing (15)–(17). For the laminar flow of a water film having a thickness of $h \sim 10^{-4}$, an average velocity of $v_w \sim 0.1$ m/sec, and $H \sim 1.5$ m the left-hand side of (15) will be equal to 0.04, i.e., inequality (17) holds. Thus, the employment of the one-dimensional approximation is legitimate. For the flow of the steam-air mixture between the shields, when $d_1 \sim 2.5 \cdot 10^{-2}$ m and $v_a = 1$ m/sec, inequalities (18) and (19) fail (the left-hand sides of expressions (16) and (17) are equal, respectively, to 10 and 14). Consequently, two-dimensional modeling of diffusion and thermal processes in the steam-air mixture between the shields is necessary and it enables one to obtain more accurate results. For this purpose one must solve two-dimensional equations of convective diffusion and heat conduction for the air flow, and the boundary conditions (temperature and density of the steam at the walls) are taken from the corresponding one-dimensional problem. In the steady-state case the diffusion equation and the boundary condition to it are written as follows:

$$v_a (y) \frac{\partial \rho}{\partial z} = \frac{\partial}{\partial y}\left(D \frac{\partial \rho}{\partial y}\right), \tag{18}$$

$$\rho \big|_{y=h} = \rho \big|_{y=d-h} = \rho_s (T_w (z)) . \tag{19}$$

The heat-conduction equation and its boundary condition have the form

$$v_a(y)\frac{\partial T_a}{\partial z} = \frac{1}{\rho_a c_a}\frac{\partial}{\partial y}\left(\lambda_a \frac{\partial T_a}{\partial y}\right), \tag{20}$$

$$T_a\big|_{y=h} = T_a\big|_{y=d-h} = T_w(z)\,. \tag{21}$$

The $v_a(y)$ profile in the two-dimensional calculations is prescribed in the laminar approximation with allowance for the adhesion condition at the boundary:

$$v_a(y) = \frac{6\,\langle v_a\rangle}{d_1^2 - 2d_1^2}y^2 - \frac{6\,\langle v_a\rangle}{d_1 - 2d_1}y\,. \tag{22}$$

The average air velocity is determined as

$$\frac{\int_0^d v_a(y)\,dy}{d} = \langle v_a\rangle\,. \tag{23}$$

The diffusion (18) and heat-conduction (20) equations with boundary conditions (19) and (21) have been solved by the "straight-line" method [9] whose essence is in obtaining the approximate solution of the boundary-value problem by replacement of the partial equation by a system of ordinary differential equations through difference approximation of the right-hand sides of (18) and (20). Upon division of the segment [0, d] into n equal parts, we obtain two systems of equations:

for the density of the steam

$$v_a(y_j)\frac{d\rho_j}{dz} = D_j\frac{\rho_{j+1} - 2\rho_j + \rho_{j-1}}{h^2}\,,\quad j = 1\ \ldots\ n\,, \tag{24}$$

for the temperature of the steam-air mixture

$$v_a(y_j)\frac{dT_j}{dz} = a_j\frac{T_{j+1} - 2T_j + T_{j-1}}{h^2}\,. \tag{25}$$

The systems of ordinary differential equations (24) and (25) were solved by the Runge–Kutta method with a fixed step in the Mathcad Professional 2000 environment, and the thickness and temperature of the film were determined at the first stage, during the solution of the boundary-value problem for the one-dimensional model [2]. Each of the systems of equations (24) and (25) contained 20 ordinary differential equations.

Calculation Results. Mathematical modeling of one special regime of operation of a film-type heat and mass exchanger enabled us to obtain the dependence of the dimensionless thickness of the water film on the dimensionless vertical coordinate (Fig. 2). The results of numerical solution of the diffusion equation (18) with boundary condition (19) are presented in Fig. 3a in the form of isolines of the density of the steam, while the results of solution of the heat-conduction equation (20) with boundary condition (21) are presented in Fig. 3b in the form of isolines of the temperature of the air between the shields.

As is seen from the figures, with upward motion between the shields the steam-air mixture becomes saturated with steam and it cools down. It is noteworthy that with such initial data the temperature of the water flowing down the shields has remained constant (T_w = const = 16.5^{o}C), since the heating of the water film by contact heat exchange is counterbalanced by its cooling in evaporation. This is precisely the essence of the special operating regime. The average temperature of the air $\langle T_a\rangle$ and the average density of the steam $\langle \rho\rangle$ at the outlet are determined from the following formulas:

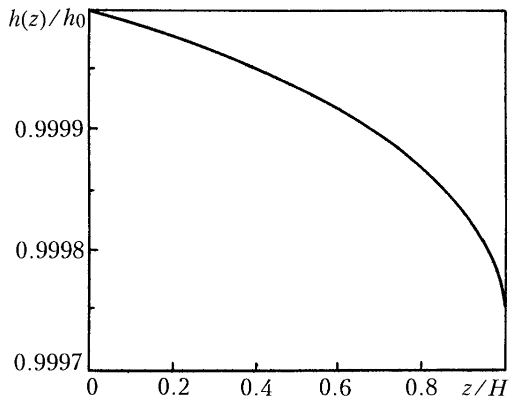

Fig. 2. Dimensionless thickness $h(z)/h_0$ of the water film vs. dimensionless vertical coordinate z/H (H = 1.5 m, d = 0.025 m, v_a = 1 m/sec, T_{w0} = 16.5°C, T_{a0} = 30°C, ψ = 25%, and Q_w = 0.1 kg/(m·sec)).

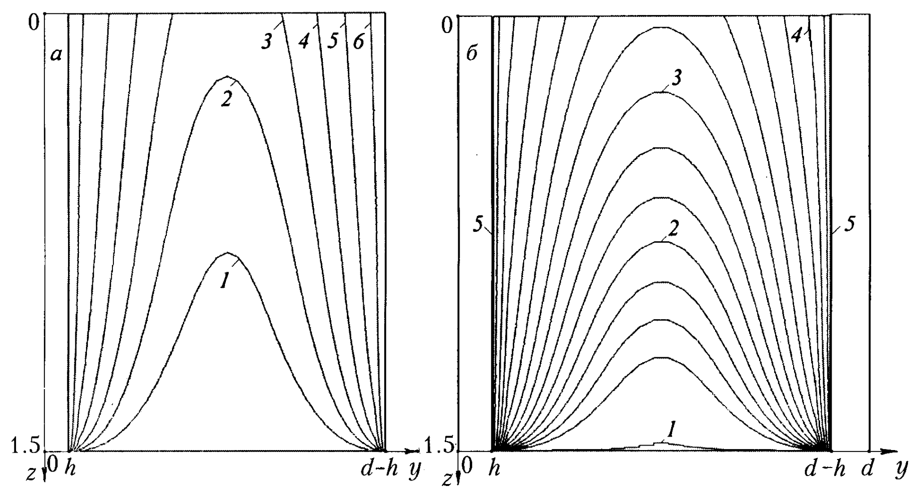

Fig. 3. Isolines of the density of the steam (a) [1) $\rho = 8.74 \cdot 10^{-3}$; 2) 0.01; 3) 0.011; 4) 0.012; 5) 0.013; 6) 0.014 kg/m^3] and the temperature of the air (b) [1) T_a = 30; 2) 25; 3) 20; 4) 18°C; 5) T_w = 16.5°C = const] between the shields. The dependences are obtained for the same conditions as those in Fig. 2.

$$\langle T_a \rangle = \frac{\int_0^d v_a(y)\, T_a(y, H)\, dy}{\int_0^d v_a(y)\, dy}, \quad \langle \rho \rangle = \frac{\int_0^d v_a(y)\, \rho(y, H)\, dy}{\int_0^d v_a(y)\, dy}. \tag{26}$$

The numerical experiments have shown that if the left-hand side of (17) is equal to 0.8, the results of the one-dimensional and two-dimensional calculations of the average density of the steam ρ coincide within 93%. If the left-hand side of (17) increases to 14, which corresponds to the parameters in Fig. 3, the absolute value of the relative difference in the results of the calculation of ρ increases to 19%. If the left-hand side of (16) increases from 0.6 to 10, the absolute value of the relative difference in the results of the calculation of the air temperature increases from 9 to 12% respectively.

The mass flow rate of the evaporated water in the humidifier mainly depends on the initial temperature of the water and the relative humidity of the steam-air mixture. For the above data the flow rate of the water evaporated Q_{evap} is approximately 0.0006 of the mass flow rate of the water in the humidifier.

CONCLUSIONS

The considered scheme of counterflow interaction of the water films on two vertical adiabatic shields and the air between them is used in the sprayers of cooling towers of electric power plants, in packed and film-type columns, evaporators, absorbers, and humidifiers widespread in the power, heavy, chemical, and food industries and in agriculture. The mathematical model proposed can also be applied to description of the processes of heat and mass exchange between films of other liquids and different gas flows, for which purpose it is necessary to change thermophysical parameters and transfer coefficients.

The algorithm of solution of the two-dimensional problem developed for determination of the parameters of a steam-air mixture enables one to more accurately calculate the parameters of the steam-air mixture in systems where criteria (16) and (17) fail (for example, in those with short shields or when the distance between them is large), although this algorithm does not employ the coefficients of heat exchange (8) and mass exchange (10).

The comparison of the results of calculations of the air parameters according to the one-dimensional (1)–(6) and two-dimensional (18)–(21) models made has shown that, as the left-hand sides of criteria (15)–(17) increase, the difference between the results of the calculation according to the first model and the average values obtained using the second model increase. The relative error can attain 20%.

The turbulent regime of flow of the steam-air mixture can also be investigated using the model proposed when the corresponding velocity profile and the turbulent coefficient of diffusion of the steam are employed, which is the subject of our further investigation.

The preliminary results of the work proposed above have been presented in [10].

We express our thanks to A. I. Shnip and the participants of his seminar for fruitful discussion of the work.

NOTATION

a, thermal diffusivity, m^2/sec; c, heat capacity, J/(kg·^{o}C); d, distance between the shields, m; d_1, cross section for passage of the air between the shields, m; D, diffusion coefficient of the steam, m^2/sec; g, free-fall acceleration, m^2/sec; H, height of the humidifier shields, m; h, thickness of the water film, m; L, characteristic distance in the transverse direction, m; r, latent heat of evaporation, J/kg; T, temperature, ^{o}C; v, velocity, m/sec; x, z, and y, longitudinal coordinates and transverse coordinate; Q_w, wetting density, kg/(m·sec); Q_{evap}, mass flow rate of the evaporated water, kg/sec; λ, thermal conductivity, W/(m·^{o}C); μ, coefficient of dynamic viscosity of the air, kg/(m·sec); ν, kinematic viscosity, m^2/sec; ρ, density, kg/m^3; $\tau_\perp$ and $\tau_\|$, characteristic time of equalization of the profile in the transverse and longitudinal directions, sec; ψ, relative humidity of the air, %; γ, mass-exchange coefficient, m/sec; α, heat-exchange coefficient, W/(m^2·^{o}C); Σ, discrepancy criterion; Re, Pr, Nu, and Sh, Reynolds, Prandtl, Nusselt, and Sherwood numbers; $\langle ... \rangle$, average value. Subscripts: s, saturated steam; a, air; w, water; 0, initial; evap, evaporated; $\perp$, transverse; $\|$, longitudinal.

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